GRADE 7 MATH TEACHING GUIDE

Lesson 7: Forms of Rational Numbers and Addition and Subtraction of Rational Numbers

Prerequisite Concepts: definition of rational numbers, subsets of real numbers, fractions, decimals

Objectives:
In this lesson, you are expected to:
1. Express rational numbers from fraction form to decimal form (terminating and repeating and non-terminating) and vice versa;
2. Add and subtract rational numbers;
3. Solve problems involving addition and subtraction of rational numbers.

NOTE TO THE TEACHER:
The first part of this module is a lesson on changing rational numbers from one form to another, paying particular attention to changing rational numbers in non-terminating and repeating decimal form to fraction form. It is assumed that students know decimal fractions and how to operate on fractions and decimals.

Lesson Proper:
A. Forms of Rational Numbers
   I. Activity
   1. Change the following rational numbers in fraction form or mixed number form to decimal form:
      a. \(-\frac{1}{4} = -0.25\)  
      d. \(\frac{5}{2} = 2.5\)  
      b. \(\frac{3}{10} = 0.3\)  
      e. \(-\frac{17}{10} = -1.7\)  
      c. \(3\frac{5}{100} = 3.05\)  
      f. \(-2\frac{1}{5} = -2.2\)

      NOTE TO THE TEACHER:
      These should be treated as review exercises. There is no need to spend too much time on reviewing the concepts and algorithms involved here.

   2. Change the following rational numbers in decimal form to fraction form.
      a. \(1.8 = \frac{9}{5}\)  
      d. \(-0.001 = \frac{-1}{1000}\)  
      b. \(-3.5 = \frac{-7}{2}\)  
      e. \(10.999 = \frac{10999}{1000}\)
c. \(-2.2 = \frac{11}{5}\)

f. \(0 \overline{11} = \frac{1}{9}\)

**NOTE TO THE TEACHER:**

The discussion that follows assumes that students remember why certain fractions are easily converted to decimals. It is not so easy to change fractions to decimals if they are not decimal fractions. Be aware of the fact that this is the time when the concept of a fraction becomes very different. The fraction that students remember as indicating a part of a whole or of a set is now a number (rational) whose parts (numerator and denominator) can be treated separately and can even be divided! This is a major shift in concept and students have to be prepared to understand how these concepts are consistent with what they know from elementary level mathematics.

**II. Discussion**

*Non-decimal Fractions*

There is no doubt that most of the above exercises were easy for you. This is because all except item 2f are what we call decimal fractions. These numbers are all parts of powers of 10. For example, \(-\frac{1}{4} = \frac{25}{100}\) which is easily convertible to a decimal form, 0.25. Likewise, the number \(-3.5 = -3 \frac{5}{10} = -\frac{35}{10}\).

What do you do when the rational number is not a decimal fraction? How do you convert from one form to the other?

Remember that a rational number is a quotient of 2 integers. To change a rational number in fraction form, you need only to divide the numerator by the denominator.

Consider the number \(\frac{1}{8}\). The smallest power of 10 that is divisible by 8 is 1000. But, \(\frac{1}{8}\) means you are dividing 1 whole unit into 8 equal parts. Therefore, divide 1 whole unit first into 1000 equal parts and then take \(\frac{1}{8}\) of the thousandths part. That is equal to \(\frac{125}{1000}\) or 0.125.
Example: Change $\frac{1}{16}$, $\frac{9}{11}$ and $-\frac{1}{3}$ to their decimal forms.

The smallest power of 10 that is divisible by 16 is 10,000. Divide 1 whole unit into 10,000 equal parts and take $\frac{1}{16}$ of the ten thousandths part. That is equal to $\frac{625}{10000}$ or 0.625. You can obtain the same value if you perform the long division $1 \div 16$.

Do the same for $\frac{9}{11}$. Perform the long division $9 \div 11$ and you should obtain $0.\overline{81}$. Therefore, $\frac{9}{11} = 0.\overline{81}$. Also, $-\frac{1}{3} = -0.\overline{3}$. Note that both $\frac{9}{11}$ and $-\frac{1}{3}$ are non-terminating but repeating decimals.

To change rational numbers in decimal forms, express the decimal part of the numbers as a fractional part of a power of 10. For example, -2.713 can be changed initially to $-2\frac{713}{1000}$ and then changed to $-\frac{2173}{1000}$.

What about non-terminating but repeating decimal forms? How can they be changed to fraction form? Study the following examples:

**Example 1**: Change $0.\overline{2}$ to its fraction form.

Solution: Let

\[ r = 0.222... \]
\[ 10r = 2.222... \]

Then subtract the first equation from the second equation and obtain

\[ 9r = 2.0 \]
\[ r = \frac{2}{9} \]

Therefore, $0.\overline{2} = \frac{2}{9}$. 

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Example 2. Change \(-1.\overline{35}\) to its fraction form.

Solution: Let

\[ r = -1.353535... \]

\[ 100r = -135.353535... \]

Then subtract the first equation from the second equation and obtain

\[ 99r = -134 \]

\[ r = -\frac{134}{99} = -1\frac{34}{99} \]

Therefore, \(-1.\overline{35}\) = \(-\frac{135}{99}\).

**NOTE TO THE TEACHER:**

Now that students are clear about how to change rational numbers from one form to another, they can proceed to learning how to add and subtract them. Students will realize soon that these skills are the same skills they learned back in elementary mathematics.

**B. Addition and Subtraction of Rational Numbers in Fraction Form**

**I. Activity**

Recall that we added and subtracted whole numbers by using the number line or by using objects in a set.

Using linear or area models, find the sum or difference.

a. \(\frac{3}{5} + \frac{1}{5} = \) _____

b. \(\frac{1}{8} + \frac{5}{8} = \) _____

c. \(\frac{10}{11} - \frac{3}{11} = \) _____

d. \(3\frac{6}{7} - 1\frac{2}{7} = \) _____

Without using models, how would you get the sum or difference?
Consider the following examples:

1. \[ \frac{1}{6} + \frac{1}{2} = \frac{1}{6} + \frac{3}{6} = \frac{4}{6} \text{ or } \frac{2}{3} \]
2. \[ \frac{6}{7} + \left( -\frac{2}{3} \right) = \frac{18}{21} + \left( -\frac{14}{21} \right) = \frac{4}{21} \]
3. \[ -\frac{4}{5} + \left( -\frac{1}{5} \right) = -\frac{20}{15} + \left( -\frac{3}{15} \right) = -\frac{23}{15} \text{ or } -1 \frac{8}{15} \]
4. \[ \frac{14}{5} - \frac{4}{7} = \frac{98}{35} - \frac{20}{35} = \frac{78}{35} \text{ or } 2\frac{8}{35} \]
5. \[ \frac{-7}{12} - \left( -\frac{2}{3} \right) = \frac{-7}{12} - \left( -\frac{8}{12} \right) = \frac{-7 + 8}{12} = \frac{1}{12} \]
6. \[ -\frac{1}{6} - \left( -\frac{11}{20} \right) = -\frac{10}{60} - \left( -\frac{33}{60} \right) = \frac{-10 + 33}{60} = \frac{23}{60} \]

Answer the following questions:

1. Is the common denominator always the same as one of the denominators of the given fractions?
2. Is the common denominator always the greater of the two denominators?
3. What is the least common denominator of the fractions in each example?
4. Is the resulting sum or difference the same when a pair of dissimilar fractions is replaced by any pair of similar fractions?

Problem: Copy and complete the fraction magic square. The sum in each row, column, and diagonal must be 2.

<table>
<thead>
<tr>
<th>a</th>
<th>1/2</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>7/5</td>
<td>1/3</td>
<td>c</td>
</tr>
<tr>
<td>d</td>
<td>e</td>
<td>2/5</td>
</tr>
</tbody>
</table>

» What are the values of a, b, c, d and e? a = \( \frac{1}{6} \), b = \( \frac{4}{3} \), c = \( \frac{4}{15} \), d = \( \frac{13}{30} \), e = \( \frac{7}{6} \)

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NOTE TO THE TEACHER:
The following pointers are not new to students at this level. However, if they had not mastered how to add and subtract fractions and decimals well, this is the time for them to do so.

Important things to remember
To Add or Subtract Fraction
• With the same denominator,
   If $a$, $b$ and $c$ denote integers, and $b \neq 0$, then
   \[
   \frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}
   \]
   and
   \[
   \frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}
   \]

• With different denominators, $\frac{a}{b}$ and $\frac{c}{d}$, where $b \neq 0$ and $d \neq 0$
   If the fractions to be added or subtracted are dissimilar
   » Rename the fractions to make them similar whose denominator is the least common multiple of $b$ and $d$.
   » Add or subtract the numerators of the resulting fractions.
   » Write the result as a fraction whose numerator is the sum or difference of the numerators and whose denominator is the least common multiple of $b$ and $d$.

Examples:
To Add:
\[
\begin{align*}
a.\quad \frac{3}{7} + \frac{2}{7} &= \frac{3+2}{7} = \frac{5}{7} \\
b.\quad \frac{2}{5} + \frac{1}{4} &= \frac{8+5}{20} = \frac{13}{20}
\end{align*}
\]
LCM/LCD of 5 and 4 is 20
\[
\begin{align*}
a.\quad \frac{4}{5} - \frac{1}{4} &= \frac{16-5}{20} = \frac{11}{20}
\end{align*}
\]
NOTE TO THE TEACHER:
Below are the answers to the activity. Make sure that students clearly understand the answers to all the questions and the concepts behind each question.

II. Questions to Ponder (Post—Activity Discussion)
Let us answer the questions posed in activity.
You were asked to find the sum or difference of the given fractions.

a. \( \frac{3}{5} + \frac{1}{5} = \frac{4}{5} \)

b. \( \frac{1}{6} + \frac{5}{8} = \frac{6}{8} \text{ or } \frac{3}{4} \)

c. \( \frac{10}{11} - \frac{5}{11} = \frac{7}{11} \)

d. \( 3 \frac{6}{7} - 1 \frac{2}{7} = 2 \frac{4}{7} \)

Without using the models, how would you get the sum or difference?
You would have to apply the rule for adding or subtracting similar fractions.

1. Is the common denominator always the same as one of the denominators of the given fractions?
   Not always. Consider \( \frac{2}{5} + \frac{3}{4} \). Their least common denominator is 20 not 5 or 4.

2. Is the common denominator always the greater of the two denominators?
   Not always. The least common denominator is always greater than or equal to one of the two denominators and it may not be the greater of the two denominators.

3. What is the least common denominator of the fractions in each example?
   (1) 6   (2) 21   (3) 15   (4) 35   (5) 12   (6) 60

4. Is the resulting sum or difference the same as when a pair of dissimilar fractions is replaced by any pair of similar fractions?
   Yes, for as long as the replacement fractions are equivalent to the original fractions.

NOTE TO THE TEACHER:
Answers in simplest form or lowest terms could mean both mixed numbers with the fractional part in simplest form or an improper fraction whose numerator and denominator have no common factor except 1. Both are acceptable as simplest forms.
### III. Exercises

Do the following exercises.

a. Perform the indicated operations and express your answer in simplest form.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(\frac{2}{9} + \frac{3}{9} + \frac{1}{5})</td>
<td>(\frac{2}{3})</td>
</tr>
<tr>
<td>2.</td>
<td>(\frac{6}{5} + \frac{3}{5} + \frac{4}{5})</td>
<td>(\frac{13}{5})</td>
</tr>
<tr>
<td>3.</td>
<td>(\frac{2}{5} + \frac{7}{10})</td>
<td>(\frac{11}{10} = \frac{1}{1})</td>
</tr>
<tr>
<td>4.</td>
<td>(\frac{16}{24} - \frac{6}{12})</td>
<td>(\frac{1}{6})</td>
</tr>
<tr>
<td>5.</td>
<td>(\frac{2\frac{5}{12}}{2} - \frac{2}{3})</td>
<td>(\frac{7}{4})</td>
</tr>
<tr>
<td>6.</td>
<td>(8\frac{1}{4} + \frac{2}{7})</td>
<td>(8\frac{15}{28} = \frac{45}{28})</td>
</tr>
<tr>
<td>7.</td>
<td>(3\frac{1}{4} + 6\frac{2}{5})</td>
<td>(9\frac{11}{12})</td>
</tr>
<tr>
<td>8.</td>
<td>(9\frac{5}{7} - 3\frac{2}{7})</td>
<td>(6\frac{3}{7})</td>
</tr>
<tr>
<td>9.</td>
<td>(\frac{7}{9} - \frac{1}{12})</td>
<td>(\frac{25}{36})</td>
</tr>
<tr>
<td>10.</td>
<td>(11\frac{5}{9} - 7\frac{5}{6})</td>
<td>(\frac{67}{18} = \frac{313}{18})</td>
</tr>
<tr>
<td>11.</td>
<td>(\frac{1}{4} + \frac{2}{3} - \frac{1}{2})</td>
<td>(\frac{5}{12})</td>
</tr>
<tr>
<td>12.</td>
<td>(10 - 3\frac{5}{11})</td>
<td>(72\frac{6}{11} = \frac{66}{11})</td>
</tr>
<tr>
<td>13.</td>
<td>(\frac{7}{20} + \frac{3}{8} + \frac{2}{5})</td>
<td>(\frac{9}{8})</td>
</tr>
<tr>
<td>14.</td>
<td>(\frac{5}{12} + \frac{4}{9} - \frac{1}{4})</td>
<td>(\frac{11}{18})</td>
</tr>
<tr>
<td>15.</td>
<td>(2\frac{5}{8} + \frac{1}{2} + 7\frac{3}{4})</td>
<td>(\frac{87}{8} = 10\frac{7}{8})</td>
</tr>
</tbody>
</table>

b. Give the number asked for.

1. What is three more than three and one-fourth? \(6\frac{1}{4}\)

NOTE TO THE TEACHER:

You should give more exercises if needed. You, the teacher should probably use the calculator to avoid computing mistakes.

Note that the language here is crucial. Students need to translate the English phrases to the correct mathematical phrase or equation.
2. Subtract from $15\frac{1}{2}$ the sum of $2\frac{1}{3}$ and $4\frac{2}{5}$. What is the result? $\frac{263}{30} = 8\frac{23}{30}$

3. Increase the sum of $6\frac{3}{14}$ and $2\frac{2}{7}$ by $3\frac{1}{2}$. What is the result? 12

4. Decrease $21\frac{3}{8}$ by $5\frac{4}{5}$. What is the result? $\frac{647}{40} = 16\frac{7}{40}$

5. What is $-8\frac{4}{5}$ minus $3\frac{2}{7}$? $-\frac{423}{35} = -12\frac{3}{35}$

c. Solve each problem.

1. Michelle and Corazon are comparing their heights. If Michelle’s height is $120\frac{3}{4}$ cm. and Corazon’s height is $96\frac{1}{3}$ cm. What is the difference in their heights? Answer: $24\frac{5}{12}$ cm

2. Angel bought $6\frac{3}{4}$ meters of silk, $3\frac{1}{2}$ meters of satin and $8\frac{2}{5}$ meters of velvet. How many meters of cloth did she buy? Answer: $18\frac{13}{20}$ m

3. Arah needs $10\frac{1}{4}$ kg. of meat to serve 55 guests. If she has $3\frac{1}{2}$ kg of chicken, a $2\frac{3}{4}$ kg of pork, and $4\frac{1}{4}$ kg of beef, is there enough meat for 55 guests? Answer: Yes, she has enough. She has a total of $10\frac{1}{2}$ kilos.

4. Mr. Tan has $13\frac{2}{5}$ liters of gasoline in his car. He wants to travel far so he added $16\frac{1}{2}$ liters more. How many liters of gasoline is in the tank? Answer: $29\frac{9}{10}$ liters

5. After boiling, the $17\frac{3}{4}$ liters of water was reduced to $9\frac{2}{3}$ liters. How much water has evaporated? Answer: $8\frac{1}{12}$ liters
NOTE TO THE TEACHER:
The last portion of this module is on the addition and subtraction of rational numbers in decimal form. This is mainly a review but emphasize that they are not just working on decimal numbers but with rational numbers. Emphasize that these decimal numbers are a result of the numerator being divided by the denominator of a quotient of two integers.

C. Addition and Subtraction of Rational Numbers in Decimal Form

There are 2 ways of adding or subtracting decimals.
1. Express the decimal numbers in fractions then add or subtract as described earlier.
   Example:
   Add: $2.3 + 7.21$
   \[
   2 \frac{3}{10} + 7 \frac{21}{100}
   \]
   \[
   (2 + 7) + \left( \frac{30 + 21}{100} \right) = 9 \frac{51}{100} = 9.51
   \]
   Subtract: $9.6 - 3.25$
   \[
   9 \frac{6}{10} - 3 \frac{25}{100}
   \]
   \[
   (9 - 3) + \frac{60 - 25}{100} = 6 \frac{35}{100} = 6.35
   \]

2. Arrange the decimal numbers in a column such that the decimal points are aligned, then add or subtract as with whole numbers.
   Example:
   Add: $2.3 + 7.21$
   \[
   \begin{array}{c}
   \hline
   & 2.3 \\
   + & 7.21 \\
   \hline
   & 9.51 \\
   \end{array}
   \]
   Subtract: $9.6 - 3.25$
   \[
   \begin{array}{c}
   \hline
   & 9.6 \\
   - & 3.25 \\
   \hline
   & 6.35 \\
   \end{array}
   \]
Exercises:

1. Perform the indicated operation.
   1) \(1,902 + 21.36 + 8.7 = 1,932.06\)
   2) \(45.08 + 9.2 + 30.545 = 84.825\)
   3) \(900 + 676.34 + 78.003 = 1,654.343\)
   4) \(0.77 + 0.9768 + 0.05301 = 1.79981\)
   5) \(5.44 – 4.97 = 0.47\)
   6) \(700 – 678.891 = 21.109\)
   7) \(7.3 – 5.182 = 2.118\)
   8) \(51.005 – 21.4591 = 29.5459\)
   9) \((2.45 + 7.89) – 4.56 = 5.78\)
   10) \((10 – 5.891) + 7.99 = 12.099\)

2. Solve the following problems:
   a. Helen had P7500 for shopping money. When she got home, she had P132.75 in her pocket. How much did she spend for shopping? **P7377.25**
   b. Ken contributed P69.25, while John and Hanna gave P56.25 each for their gift to Teacher Daisy. How much were they able to gather altogether? **P181.75**
   c. Ryan said, “I’m thinking of a number N. If I subtract 10.34 from N, the difference is 1.34.” What was Ryan’s number? **11.68**
   d. Agnes said, “I’m thinking of a number N. If I increase my number by 56.2, the sum is 14.62.” What was Agnes number? **–41.58**
   e. Kim ran the 100-meter race in 135.46 seconds. Tyron ran faster by 15.7 seconds. What was Tyron’s time for the 100-meter dash? **119.76**

NOTE TO THE TEACHER:
The summary is important especially because this is a long module. This lesson provided students with plenty of exercises to help them master addition and subtraction of rational numbers.

SUMMARY
This lesson began with some activities and instruction on how to change rational numbers from one form to another and proceeded to discuss addition and subtraction of rational numbers. The exercises given were not purely computational. There were thought questions and problem solving activities that helped in deepening one’s understanding of rational numbers.