Lesson 15: Measurement and Measuring Length  

Prerequisite Concepts: Real Numbers and Operations

Objective
At the end of the lesson, you should be able to:
1. Describe what it means to measure;
2. Describe the development of measurement from the primitive to the present international system of unit;
3. Estimate or approximate length;
4. Use appropriate instruments to measure length;
5. Convert length measurement from one unit to another, including the English system;
6. Solve problems involving length, perimeter and area.

NOTE TO THE TEACHER:
This is a lesson on the English and Metric System of Measurement and using these systems to measure length. Since these systems are widely used in our community, a good grasp of this concept will help your students be more accurate in dealing with concepts involving length such as distance, perimeter and area. This lesson on measurement tackles concepts which your students have most probably encountered and will continue to deal with in their daily lives. Moreover, concepts and skills related to measurement are prerequisites to topics in Geometry as well as Algebra.

Lesson Proper
A.  
I. Activity:
Instructions: Determine the dimension of the following using only parts of your arms. Record your results in the table below. Choose a classmate and compare your results.

<table>
<thead>
<tr>
<th>SHEET OF INTERMEDIATE PAPER</th>
<th>TEACHER’S TABLE</th>
<th>CLASSROOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm part used*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comparison to: (classmate’s name)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
* For the arm part, please use any of the following only: the palm, the handspan and the forearm length

**Important Terms to Remember:**
> palm – the width of one’s hand excluding the thumb
> handspan – the distance from the tip of the thumb to the tip of the little finger of one’s hand with fingers spread apart.
> forearm length – the length of one’s forearm: the distance from the elbow to the tip of the middle finger.

**NOTE TO THE TEACHER:**
The activities in this module involve measurement of actual objects and lengths found inside the classroom but you may modify the activity and include objects and distances outside the classroom. Letting the students use non-standard units of measurement first will give them the opportunity to appreciate our present measuring tools by emphasizing on the discrepancy of their results vis-a-vis their partner’s results.

Answer the following questions:
1. What was your reason for choosing which arm part to use? Why?
2. Did you experience any difficulty when you were doing the actual measuring?
3. Were there differences in your data and your classmate’s data? Were the differences significant? What do you think caused those differences?

**II. Questions to Ponder (Post-Activity Discussion)**
Let us answer the questions in the opening activity:
1. What is the appropriate arm part to use in measuring the length and width of the sheet of paper? of the teacher’s table? Of the classroom? What was your reason for choosing which arm part to use? Why?
   ➢ While all of the units may be used, there are appropriate units of measurement to be used depending on the length you are trying to measure.
   ➢ For the sheet of paper, the palm is the appropriate unit to use since the handspan and the forearm length is too long.
   ➢ For the teacher’s table, either the palm or the handspan will do but the forearm length might be too long to get an accurate measurement.
   ➢ For the classroom, the palm and handspan may be used but you may end up with a lot of repetitions. The best unit to use would be the forearm length.
2. Did you experience any difficulty when you were doing the actual measuring?
   The difficulties you may have experienced might include having to use too many repetitions.
3. Were there differences in your data and your classmate’s data? Were the differences significant? What do you think caused those differences?
   If you and your partner vary a lot in height, then chances are your forearm length, handspan and palm may also vary, leading to different measurements of the same thing.
NOTE TO THE TEACHER:

This is a short introduction to the History of Measurement. Further research would be needed to widen the coverage of the concept. The questions that follow will help enrich the discussion on this particular topic.

**History of Measurement**

One of the earliest tools that human beings invented was the unit of measurement. In olden times, people needed measurement to determine how long or wide things are; things they needed to build their houses or make their clothes. Later, units of measurement were used in trade and commerce. In the 3rd century BC Egypt, people used their body parts to determine measurements of things; the same body parts that you used to measure the assigned things to you.

The forearm length, as described in the table below, was called a cubit. The handspan was considered a half cubit while the palm was considered 1/6 of a cubit. Go ahead, check out how many handspans your forearm length is. The Egyptians came up with these units to be more accurate in measuring different lengths.

However, using these units of measurement had a disadvantage. Not everyone had the same forearm length. Discrepancies arose when the people started comparing their measurements to one another because measurements of the same thing differed, depending on who was measuring it. Because of this, these units of measurement are called non-standard units of measurement which later on evolved into what is now the inch, foot and yard, basic units of length in the English system of measurement.

### III. Exercise:

1. Can you name other body measurements which could have been used as a non-standard unit of measurement? Do some research on other non-standard units of measurement used by people other than the Egyptians.
2. Can you relate an experience in your community where a non-standard unit of measurement was used?

### B. I. Activity

NOTE TO THE TEACHER:

In this activity, comparisons of their results will underscore the advantages of using standard units of measurement as compared to using non-standard units of measurement. However, this activity may also provide a venue to discuss the limitations of actual measurements. Emphasize on the differences of their results, however small they may be.

Instructions: Determine the dimension of the following using the specified English units only. Record your results in the table below. Choose a classmate and compare your results.
<table>
<thead>
<tr>
<th>SHEET OF INTERMEDIATE PAPER</th>
<th>TEACHER’S TABLE</th>
<th>CLASSROOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>Width</td>
<td>Length</td>
</tr>
<tr>
<td>Unit used*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comparison to:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(classmate’s name)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the unit used, choose which of the following SHOULD be used: inch or foot.

Answer the following questions:
1. What was your reason for choosing which unit to use? Why?
2. Did you experience any difficulty when you were doing the actual measuring?
3. Were there differences in your data and your classmate’s data? Were the differences as big as the differences when you used non-standard units of measurement? What do you think caused those differences?

II. Questions to Ponder (Post-Activity Discussion)
Let us answer the questions in the activity above:
1. What was your reason for choosing which unit to use? Why?
   - For the sheet of paper, the appropriate unit to use is inches since its length and width might be shorter than a foot.
   - For the table and the classroom, a combination of both inches and feet may be used for accuracy and convenience of not having to deal with a large number.
2. What difficulty, if any, did you experience when you were doing the actual measuring?
3. Were there differences in your data and your classmate’s data? Were the differences as big as the differences when you used non-standard units of measurement? What do you think caused those differences?
   - If you and your partner used the steel tape correctly, both your data should have little or no difference at all. The difference should not be as big or as significant as the difference when non-standard units of measurement were used. The slight difference might be caused by how accurately you tried to measure each dimension or by how you read the ticks on the steel tape. In doing actual measurement, a margin of error should be considered.

NOTE TO THE TEACHER:

AUTHOR: Rhett Anthony C. Latonio
Grade 7 Math LESSON 15: MEASUREMENT AND MEASURING LENGTH

The narrative that follows provides continuity to the development of the English system of measurement. The conversion factors stated herein only involve common units of length. Further research may include other English units of length.

**History of Measurement (Continued)**

As mentioned in the first activity, the inch, foot, and yard are said to be based on the cubit. They are the basic units of length of the English System of Measurement, which also includes units for mass, volume, time, temperature, and angle. Since the inch and foot are both units of length, each can be converted into the other. Here are the conversion factors, as you may recall from previous lessons:

1 foot = 12 inches
1 yard = 3 feet

For long distances, the mile is used:
1 mile = 1,760 yards = 5,280 feet

Converting from one unit to another might be tricky at first, so an organized way of doing it would be a good starting point. As the identity property of multiplication states, the product of any value and 1 is the value itself. Consequently, dividing a value by the same value would be equal to one. Thus, dividing a unit by its equivalent in another unit is equal to 1. For example:

1 foot / 12 inches = 1
3 feet / 1 yard = 1

These conversion factors may be used to convert from one unit to another. Just remember that you’re converting from one unit to another so cancelling same units would guide you in how to use your conversion factors. For example:

1. Convert 36 inches into feet:

\[
36 \text{ inches} \times \frac{1 \text{ feet}}{12 \text{ inches}} = 3 \text{ feet}
\]

2. Convert 2 miles into inches:

\[
2 \text{ miles} \times \frac{5,280 \text{ feet}}{1 \text{ mile}} \times \frac{12 \text{ inches}}{1 \text{ foot}} = \frac{2 \times 5,280 \times 12}{1 \times 1} \text{ inches} = 126,720 \text{ inches}
\]

Again, since the given measurement was multiplied by conversion factors which are equal to 1, only the unit was converted but the given length was not changed.

Try it yourself.

**III. Exercise:**

Convert the following lengths into the desired unit:

1. Convert 30 inches to feet
   
   **Solution:** \(30 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}} = 2.5 \text{ feet}\)

2. Convert 130 yards to inches
   
   **Solution:** \(130 \text{ yards} \times \frac{3 \text{ feet}}{1 \text{ yard}} \times \frac{12 \text{ inches}}{1 \text{ foot}} = 4,680 \text{ inches}\)

AUTHOR: Rhett Anthony C. Latonio
3. Sarah is running in a 42-mile marathon. How many more feet does Sarah need to run if she has already covered 64,240 yards?

Solution:

Step 1: 
\[42 \text{ miles} \times \frac{5280 \text{ feet}}{1 \text{ mile}} = 221,760 \text{ feet}\]

Step 2: 
\[64,240 \text{ yards} \times \frac{3 \text{ feet}}{1 \text{ yard}} = 192,720 \text{ feet}\]

Step 3: 
\[221,760 \text{ feet} - 192,720 \text{ feet} = 29,040 \text{ feet}\]

Answer: Sarah needs to run 29,040 feet to finish the marathon

NOTE TO TEACHER:

In item 3, disregarding the units and not converting the different units of measurement into the same units of measurement is a common error.

C. I. Activity:

NOTE TO THE TEACHER:

This activity introduces the metric system of measurement and its importance. This also highlights how events in Philippine and world history determined the systems of measurement currently used in the Philippines.

Answer the following questions:

1. When a Filipina girl is described as 1.7 meters tall, would she be considered tall or short? How about if the Filipina girl is described as 5 ft, 7 inches tall, would she be considered tall or short?
2. Which particular unit of height were you more familiar with? Why?

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions in the activity above:

1. When a Filipina girl is described as 1.7 meters tall, would she be considered tall or short? How about if the Filipina girl is described as 5 ft, 7 inches tall, would she be considered tall or short?
   - Chances are, you would find it difficult to answer the first question. As for the second question, a Filipina girl with a height of 5 feet, 7 inches would be considered tall by Filipino standards.

2. Which particular unit of height were you more familiar with? Why?
   - Again, chances are you would be more familiar with feet and inches since feet and inches are still being widely used in measuring and describing height here in the Philippines.
NOTE TO THE TEACHER:

The reading below discusses the development of the Metric system of measurement and the prefixes which the students may use or may encounter later on. Further research may include prefixes which are not commonly used as well as continuing efforts in further standardization of the different units.

History of Measurement (Continued)

The English System of Measurement was widely used until the 1800s and the 1900s when the Metric System of Measurement started to gain ground and became the most used system of measurement worldwide. First described by Belgian Mathematician Simon Stevin in his booklet, De Thiende (The Art of Tenths) and proposed by English philosopher, John Wilkins, the Metric System of Measurement was first adopted by France in 1799. In 1875, the General Conference on Weights and Measures (Conférence générale des poids et mesures or CGPM) was tasked to define the different measurements. By 1960, CGPM released the International System of Units (SI) which is now being used by majority of the countries with the biggest exception being the United States of America. Since our country used to be a colony of the United States, the Filipino people were schooled in the use of the English instead of the Metric System of Measurement. Thus, the older generation of Filipinos is more comfortable with English System rather than the Metric System although the Philippines have already adopted the Metric System as its official system of measurement.

The Metric System of Measurement is easier to use than the English System of Measurement since its conversion factors would consistently be in the decimal system, unlike the English System of Measurement where units of lengths have different conversion factors. Check out the units used in your steep tape measure, most likely they are inches and centimeters. The base unit for length is the meter and units longer or shorter than the meter would be achieved by adding prefixes to the base unit. These prefixes may also be used for the base units for mass, volume, time and other measurements. Here are the common prefixes used in the Metric System:

<table>
<thead>
<tr>
<th>PREFIX</th>
<th>SYMBOL</th>
<th>FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>tera</td>
<td>T</td>
<td>x 1,000,000,000,000</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
<td>x 1,000,000,000</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>x 1,000,000</td>
</tr>
<tr>
<td>kilo</td>
<td>k</td>
<td>x 1,000</td>
</tr>
<tr>
<td>hecto</td>
<td>h</td>
<td>x 100</td>
</tr>
<tr>
<td>deka</td>
<td>da</td>
<td>x 10</td>
</tr>
<tr>
<td>deci</td>
<td>d</td>
<td>x 1/10</td>
</tr>
<tr>
<td>centi</td>
<td>c</td>
<td>x 1/100</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>x 1/1,000</td>
</tr>
<tr>
<td>micro</td>
<td>µ</td>
<td>x 1/1,000,000</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>x 1/1,000,000,000</td>
</tr>
</tbody>
</table>
For example:
1 kilometer = 1,000 meters
1 millimeter = 1/1,000 meter or 1,000 millimeters = 1 meter

These conversion factors may be used to convert from big to small units or vice versa. For example:
1. Convert 3 km to m:
\[3 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} = 3000 \text{ m}\]

2. Convert 10 mm to m:
\[10 \text{ mm} \times \frac{1 \text{ m}}{1000 \text{ mm}} = \frac{1}{100} \text{ or } 0.01 \text{ m}\]

As you can see in the examples above, any length or distance may be measured using the appropriate English or Metric units. In the question about the Filipina girl whose height was expressed in meters, her height can be converted to the more familiar feet and inches. So, in the Philippines where the official system of measurements is the Metric System yet the English System continues to be used, or as long as we have relatives and friends residing in the United States, knowing how to convert from the English System to the Metric System (or vice versa) would be useful. The following are common conversion factors for length:

1 inch = 2.54 cm
3.3 feet ≈ 1 meter

For example:
Convert 20 inches to cm:
\[20 \text{ in} \times \frac{2.54 \text{ cm}}{1 \text{ in}} = 50.8 \text{ cm}\]

III. Exercise:

NOTE TO THE TEACHER:
Knowing the lengths of selected body parts will help students in estimating lengths and distances by using these body parts and their measurements to estimate certain lengths and distances. Items 5 & 6 might require a review in determining the perimeter and area of common geometric figures.

1. Using the tape measure, determine the length of each of the following in cm. Convert these lengths to meters.
Grade 7 Math LESSON 15: MEASUREMENT AND MEASURING LENGTH

<table>
<thead>
<tr>
<th>Centimeters</th>
<th>Meters</th>
</tr>
</thead>
</table>

2. Using the data in the table above, estimate the lengths of the following without using the steel tape measure or ruler:

<table>
<thead>
<tr>
<th>BALLPEN</th>
<th>LENGTH OF WINDOW PANES</th>
<th>LENGTH OF YOUR FOOT FROM THE TIP OF YOUR HEEL TO THE TIP OF YOUR TOES</th>
<th>HEIGHT OF THE CHALKBOARD</th>
<th>LENGTH OF THE CHALKBOARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>NON-STANDARD UNIT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>METRIC UNIT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Using the data from table 1, convert the dimensions of the sheet of paper, teacher’s table and the classroom into Metric units. Recall past lessons on perimeter and area and fill in the appropriate columns:

<table>
<thead>
<tr>
<th>SHEET OF INTERMEDIATE PAPER</th>
<th>TEACHER’S TABLE</th>
<th>CLASSROOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>Width</td>
<td>Length</td>
</tr>
<tr>
<td>English units</td>
<td></td>
<td>Metric</td>
</tr>
</tbody>
</table>

4. Two friends, Zale and Enzo, run in marathons. Zale finished a 21-km marathon in Cebu while Enzo finished a 15-mile marathon in Los Angeles. Who between the two ran a longer distance? By how many meters?

\[
\text{Step 1: } 21 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} = 21,000 \text{ m}
\]

\[
\text{Step 2: } 15 \text{ mi} \times \frac{1.6 \text{ km}}{1 \text{ mi}} \times \frac{1000 \text{ m}}{1 \text{ km}} = 24,000 \text{ m}
\]

\[
\text{Step 3: } 24,000 \text{ m} – 21,000 \text{ m} = 3,000 \text{ m}
\]

\text{Answer: Enzo ran a distance of 3,000 meters more.}

AUTHOR: Rhett Anthony C. Latonio
5. Georgia wants to fence her square garden, which has a side of 20 feet, with two rows of barb wire. The store sold barb wire by the meter at P12/meter. How much money will Georgia need to buy the barb wire she needs?

*Step 1:* 20 ft x 4 sides x 2 rows = 160 ft

*Step 2:* 160 ft x \( \frac{1 \text{ m}}{3.3 \text{ ft}} \) = 48.48 m rounded up to 49 m since the store sells barb wire by the m

*Step 3:* 49 m x P12/meter = P 588

**Answer:** Georgia will need P 588 to buy 49 meters of barb wire

5. A rectangular room has a floor area of 32 square meters. How many tiles, each measuring 50 cm x 50 cm, are needed to cover the entire floor?

*Step 1:* 50 cm x \( \frac{1 \text{ m}}{100 \text{ cm}} \) = 0.5 m

*Step 2:* Area of 1 tile: 0.5 m x 0.5 m = 0.25 m^2

*Step 3:* 32 m^2 / 0.25 m^2 = 128 tiles

**Answer:** 128 tiles are needed to cover the entire floor

**Summary**

In this lesson, you learned: 1) that ancient Egyptians used units of measurement based on body parts such as the cubit and the half cubit. The cubit is the length of the forearm from the elbow to the tip of the middle finger; 2) that the inch and foot, the units for length of the English System of Measurement, are believed to be based on the cubit; 3) that the Metric System of Measurement became the dominant system in the 1900s and is now used by most of the countries with a few exceptions, the biggest exception being the United States of America; 4) that it is appropriate to use short base units of length for measuring short lengths and long units of lengths to measure long lengths or distances; 5) how to convert common English units of length into other English units of length using conversion factors; 6) that the Metric System of Measurement is based on the decimal system and is therefore easier to use; 7) that the Metric System of Measurement has a base unit for length (meter) and prefixes to signify long or short lengths or distances; 8) how to estimate lengths and distances using your arm parts and their equivalent Metric lengths; 9) how to convert common Metric units of length into other Metric units of length using the conversion factors based on prefixes; 10) how to convert common English units of length into Metric units of length (and vice versa) using conversion factors; 11) how to solve length, perimeter and area problems using English and Metric units.