Lesson 3: Problems Involving Sets

Prerequisite Concepts: Operations on Sets and Venn Diagrams

Objectives:
In this lesson, you are expected to:
1. Solve word problems involving sets with the use of Venn diagrams
2. Apply set operations to solve a variety of word problems.

NOTE TO THE TEACHER
This is an important lesson. Do not skip it. This lesson reinforces what students learned about sets, set operations and the Venn diagram in solving problems.

Lesson Proper:
I. Activity
Try solving the following problem:
In a class of 40 students, 17 have ridden an airplane, 28 have ridden a boat. 10 have ridden a train, 12 have ridden both an airplane and a boat. 3 have ridden a train only and 4 have ridden an airplane only. Some students in the class have not ridden any of the three modes of transportation and an equal number have taken all three.
   a. How many students have used all three modes of transportation?
   b. How many students have taken only the boat?

NOTE TO THE TEACHER
Allow students to write their own solutions. Allow them to discuss and argue. In the end, you have to know how to steer them to the correct solution.

II. Questions/Points to Ponder (Post-Activity Discussion)
Venn diagrams can be used to solve word problems involving union and intersection of sets. Here are some worked out examples:
1. A group of 25 high school students were asked whether they use either Facebook or Twitter or both. Fifteen of these students use Facebook and twelve use Twitter.
   a. How many use Facebook only?
b. How many use Twitter only?
c. How many use both social networking sites?

Solution:

Let $S_1 =$ set of students who use Facebook only
$S_2 =$ set of students who use both social networking sites
$S_3 =$ set of students who use Twitter only

The Venn diagram is shown below:

Finding the elements in each region:

\[
\begin{align*}
 n(S_1) + n(S_2) + n(S_3) &= 25 \\
 n(S_1) + n(S_2) &= 15 \\
\frac{n(S_3) = 10}{n(S_1)} = 13 \\
\text{But } n(S_2) + n(S_3) &= 12 \\
\frac{n(S_2) = 2}{n(S_3)}
\end{align*}
\]

The number of elements in each region is shown below:

\[
\begin{array}{c}
\text{Facebook} \\
\text{Twitter}
\end{array}
\begin{array}{c}
13 \\
2 \\
10
\end{array}
\]
2. A group of 50 students went in a tour in Palawan province. Out of the 50 students, 24 joined the trip to Coron; 18 went to Tubbataha Reef; 20 visited El Nido; 12 made a trip to Coron and Tubbataha Reef; 15 saw Tubbataha Reef and El Nido; 11 made a trip to Coron and El Nido and 10 saw the three tourist spots.
   a. How many of the students went to Coron only?
   b. How many of the students went to Tubbataha Reef only?
   c. How many joined the El Nido trip only?
   d. How many did not go to any of the tourist spots?

Solution:
To solve this problem, let

$P_1 = \text{students who saw the three tourist spots}$
$P_2 = \text{those who visited Coron only}$
$P_3 = \text{those who saw Tubbataha Reef only}$
$P_4 = \text{those who joined the El Nido trip only}$
$P_5 = \text{those who visited Coron and Tubbataha Reef only}$
$P_6 = \text{those who joined the Tubbataha Reef and El Nido trip only}$
$P_7 = \text{those who saw Coron and El Nido only}$
$P_8 = \text{those who did not see any of the three tourist spots}$

Draw the Venn diagram as shown below and identify the region where the students went.
Determine the elements in each region starting from \( P_1 \).
\( P_1 \) consists of students who went to all 3 tourist spots. Thus, \( n(P_1) = 10 \).
\( P_1 \cup P_5 \) consists of students who visited Coron and Tubbataha Reef but this set includes those who also went to El Nido. Therefore, 
\[ n(P_5) = 12 - 10 = 2 \] students visited Coron and Tubbataha Reef only.
\( P_1 \cup P_6 \) consists of students who went to El Nido and Tubbataha Reef but this set includes those who also went to Coron. Therefore, 
\[ n(P_6) = 15 - 10 = 5 \] students visited El Nido and Tubbataha Reef only.
\( P_1 \cup P_7 \) consists of students who went to Coron and El Nido but this set includes those who also went to Tubbataha Reef. Therefore, 
\[ n(P_7) = 11 - 10 = 1 \] student visited Coron and El Nido only.
From here, it follows that
\[ n(P_2) = 24 - n(P_1) - n(P_5) - n(P_7) = 24 - 10 - 2 - 1 = 11 \] students visited Coron only.
\[ n(P_3) = 18 - n(P_1) - n(P_5) - n(P_6) = 18 - 10 - 2 - 5 = 1 \] student visited Tubbataha Reef only
\[ n(P_4) = 20 - n(P_1) - n(P_6) - n(P_7) = 20 - 10 - 5 - 1 = 4 \] students visited Coron and El Nido only.
Therefore
\[ n(P_8) = 50 - n(P_1) - n(P_2) - n(P_3) - n(P_4) - n(P_5) - n(P_6) - n(P_7) = 16 \] students did not visit any of the three spots.

The number of elements is shown below.

Now, what about the opening problem? Solution to the Opening Problem (Activity):
Can you explain the numbers?

**NOTE TO THE TEACHER**
Discuss the solution thoroughly and clarify all questions your students might have. Emphasize the notation for the cardinality of a set.
III. Exercises

Do the following exercises. Represent the sets and draw a Venn diagram when needed.

1. If $A$ is a set, give two subsets of $A$. **Answer: $\emptyset$ and $A$**

2. (a) If $A$ and $B$ are finite sets and $A \subseteq B$, what can you say about the cardinalities of the two sets?
   
   (b) If the cardinality of $A$ is less than the cardinality of $B$, does it follow that $A \subseteq B$?
   
   **Answer: (a) $n(A) < n(B)$; (b) No. Example: $A = \{1, 2\}, B = \{2, 4, 6\}$**

3. If $A$ and $B$ have the same cardinality, does it follow that $A = B$? Explain.
   
   **Answer: Not necessarily. Example, $A = \{1, 2, 3\}$ and $B = \{4, 8, 9\}$**.

4. If $A \subseteq B$ and $B \subseteq C$. Does it follow that $A \subseteq C$? Illustrate your reasoning using a Venn diagram. **Answer: Yes.**

5. Among the 70 kids in Barangay Magana, 53 like eating in Jollibee while 42 like eating in McDonalds. How many like eating both in Jollibee and in McDonalds? In Jollibee only? In McDonalds only?

   **Solution:**
   
   Let $n(M_1)$ = kids who like Jollibee only
   
   $n(M_2)$ = kids who like both Jollibee and McDonalds
   
   $n(M_3)$ = kids who like McDonalds only

   **Draw the Venn diagram**
Find the elements in each region

\[ n(M_1) + n(M_2) + n(M_3) = 70 \]
\[ n(M_1) + n(M_2) = 53 \]
\[ n(M_3) = 17 \]
\[ But \quad n(M_2) + n(M_3) = 42 \]
\[ n(M_2) = 25 \]

Check using Venn diagram

6. The following diagram shows how all the First Year students of Maningning High School go to school.
a. How many students ride in a car, jeep and the MRT going to their school? \(15\)
b. How many students ride in both a car and a jeep? \(34\)
c. How many students ride in both a car and the MRT? \(35\)
d. How many students ride in both a jeep and the MRT? \(32\)
e. How many students go to school in a car only \(55\) a jeep only \(76\) in the MRT only \(67\) walking \(100\)
f. How many students First Year students of Maningning High School are there? \(269\)

7. The blood-typing system is based on the presence of proteins called antigens in the blood. A person with antigen A has blood type A. A person with antigen B has blood type B, and a person with both antigens A and B has blood type AB. If no antigen is present, the blood type is O. Draw a Venn diagram representing the ABO System of blood typing.

A protein that coats the red blood cells of some persons was discovered in 1940. A person with the protein is classified as Rh positive (Rh+), and a person whose blood cells lack the protein is Rh negative (Rh–). Draw a Venn diagram illustrating all the blood types in the ABO System with the corresponding Rh classifications.

**NOTE TO THE TEACHER**
The second problem is quite complex. Adding the 3rd set Rh captures the system without altering the original diagram in the first problem.

**Summary**
In this lesson, you were able to apply what you have learned about sets, the use of a Venn diagram and set operations in solving word problems.