Lesson 2.1: Union and Intersection of Sets

Pre-requisite Concepts: Whole Numbers, definition of sets, Venn diagrams

About the Lesson:

After learning some introductory concepts about sets, a lesson on set operations follows. The student will learn how to combine sets (union) and how to determine the elements common to 2 or 3 sets (intersection).

Objectives:

In this lesson, you are expected to:

1. Describe and define
   a. union of sets;
   b. intersection of sets.

2. Perform the set operations
   a. union of sets;
   b. intersection of sets.

3. Use Venn diagrams to represent the union and intersection of sets.

Lesson Proper:

I. Activities

Answer the following questions:

1. Which of the following shows the union of set A and set B? How many elements are in the union of A and B?
2. Which of the following shows the intersection of set A and set B? How many elements are there in the intersection of A and B?

Here’s another activity:
Let
\[ V = \{2x \mid x \in \mathbb{I}, 1 \leq x \leq 4\} \]
\[ W = \{x^2 \mid x \in \mathbb{I}, -2 \leq x \leq 2\} \]

What elements may be found in the intersection of V and W? How many are there? What elements may be found in the union of V and W? How many are there?

Do you remember how to use Venn Diagrams? Based on the diagram below, (1) determine the elements that belong to both A and B; (2) determine the elements that belong to A or B or both. How many are there in each set?

![Venn Diagram]

**Important Terms/Symbols to Remember**
The following are terms that you must remember from this point on.

1. Let A and B be sets. The *union* of the sets A and B, denoted by \( A \cup B \), is the set that contains those elements that belong to A, B, or to both.

An element \( x \) belongs to the union of the sets A and B if and only if \( x \) belongs to A or \( x \) belongs to B or to both. This tells us that
LESSON 2.1. UNION AND INTERSECTION OF SETS

A \cup B = \{ x \mid x \in A \text{ or } x \in B \}

Using the Venn diagram, all of the set of A and of B are shaded to show A \cup B.

Using the Venn diagram, all of the set of A and of B are shaded to show A \cup B.

2. Let A and B be sets. The intersection of the sets A and B, denoted by A \cap B, is the set containing those elements that belong to both A and B.

   An element x belongs to the intersection of the sets A and B if and only if x belongs to A and x belongs to B. This tells us that
   
   A \cap B = \{ x \mid x \in A \text{ and } x \in B \}

Using the Venn diagram, the set A \cap B consists of the shared regions of A and B.

Sets whose intersection is an empty set are called disjoint sets.

3. The cardinality of the union of two sets is given by the following equation:

   \[ n (A \cup B) = n (A) + n (B) - n (A \cap B). \]

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions posed in the opening activity.

1. Which of the following shows the union of set A and set B? Set 2. This is because it contains all the elements that belong to A or B or both. There are 8 elements.

2. Which of the following shows the intersection of set A and set B? Set 3. This is because it contains all elements that are in both A and B. There are 3 elements.

In the second activity:

V = \{ 2, 4, 6, 8 \}
W = \{ 0, 1, 4 \}

Therefore, \( V \cap W = \{ 4 \} \) has 1 element and \( V \cup W = \{ 0, 1, 2, 4, 6, 8 \} \) has 6 elements. Note that the element \( \{ 4 \} \) is counted only once.

On the Venn Diagram: (1) The set that contains elements that belong to both A and B consists of two elements \( \{ 1, 12 \} \); (2) The set that contains elements that belong to A or B or both consists of 6 elements \( \{ 1, 10, 12, 20, 25, 36 \} \).

### III. Exercises

1. Given sets A and B,

<table>
<thead>
<tr>
<th>Set A Students who play the guitar</th>
<th>Set B Students who play the piano</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethan Molina</td>
<td>Mayumi Torres</td>
</tr>
<tr>
<td>Chris Clemente</td>
<td>Janis Reyes</td>
</tr>
<tr>
<td>Angela Dominguez</td>
<td>Chris Clemente</td>
</tr>
<tr>
<td>Mayumi Torres</td>
<td>Ethan Molina</td>
</tr>
<tr>
<td>Joanna Cruz</td>
<td>Nathan Santos</td>
</tr>
</tbody>
</table>

determine which of the following shows (a) \( A \cup B \); and (b) \( A \cap B \)?

<table>
<thead>
<tr>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Set 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethan Molina</td>
<td>Mayumi Torres</td>
<td>Mayumi Torres</td>
<td>Ethan Molina</td>
</tr>
<tr>
<td>Chris Clemente</td>
<td>Ethan Molina</td>
<td>Janis Reyes</td>
<td>Chris Clemente</td>
</tr>
<tr>
<td>Angela Dominguez</td>
<td>Chris Clemente</td>
<td>Ethan Molina</td>
<td>Angela Dominguez</td>
</tr>
<tr>
<td>Mayumi Torres</td>
<td>Ethan Molina</td>
<td>Nathan Santos</td>
<td>Mayumi Torres</td>
</tr>
<tr>
<td>Joanna Cruz</td>
<td>Nathan Santos</td>
<td>Joanna Cruz 3</td>
<td>Joanna Cruz</td>
</tr>
</tbody>
</table>

2. Do the following exercises. Write your answers on the spaces provided:

\[
A = \{0, 1, 2, 3, 4\} \quad B = \{0, 2, 4, 6, 8\} \quad C = \{1, 3, 5, 7, 9\}
\]

Given the sets above, determine the elements and cardinality of:

a. \( A \cup B = \) ________________

b. \( A \cup C = \) ________________

c. \( A \cup B \cup C = \) ________________
d. $A \cap B = \text{______________}$

e. $B \cap C = \text{______________}$

f. $A \cap B \cap C = \text{______________}$

g. $(A \cap B) \cup C = \text{______________}$

3. Let $W = \{ \ x \ | \ 0 < x < 3 \ \}$, $Y = \{ \ x \ | \ x > 2 \ \}$, and $Z = \{ \ x \ | \ 0 \leq x \leq 4 \ \}$.

Determine (a) $(W \cup Y) \cap Z$; (b) $W \cap Y \cap Z$.

**Summary**

In this lesson, you learned the definition of union and intersection of sets. You also learned how to use Venn diagram to represent the union and the intersection of sets. You also learned how to determine the elements that belong to the union and intersection of sets.